Modelling a Brushless DC Motor Power Source Based Two-Finger Gripper

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Abstract. In industrial automation system and robotic fields, men use gripper as end manipulator to do simple manipulation tasks as grasping, screwing, pulling, etc. In order to obtain stability in manipulating an object, the gripper has to receive the proper current to produce proper gripper force. Therefore the modelling the gripper, hence, plays an important role. The gripper being modelled has two state: open and close. The finger is open, when finger moves away from each other, and close when the finger moves to each other. Opening the gripper can be done by sending voltage 0 to +5 V to the gripper, and close by sending –5 V to 0 V. The voltage magnitude determines the acceleration of the finger movement, hence, the gripforce is determined by the dynamic of the current sent to the brushless dc motor in the gripper. This paper discuss the modelling process of the gripper. At the end, we will have a mathematical model of the gripper.

Keywords : *Brushless DC Motor, modelling, identification, gripper*

1 Introduction

In order to design a good controller for a manipulation system, we need a well-modeled grasping system, which is then used to determine which controller is used[1]. The power source of the gripper is a brushless DC motor. In order to drive the motor, a PWM (Pulse Width Modulation) is used. This PWM is built in the gripper. The mechanical scheme of the finger is given below.

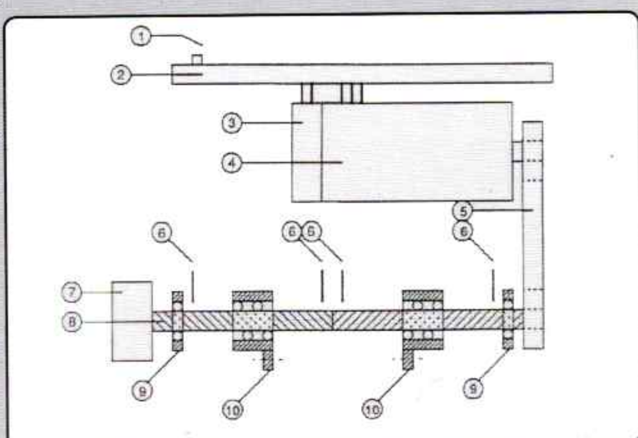

Figure 1: The scheme of the two finger parallel gripper. 1: 25 poles sub D connector, 2: PWM Servo Amplifier, 3: incremental encoder, 4: brushless DC motor, 5: belt, 6: approaching sensors, 7: electromagnetic break, 8: ball bearings rotating spindle] 9: bearings, 10: finger-fastened blockabel 4 Skala Penilaian Performansi Perusahaan

Brushless DC motor is a permanent magnet rotor DC motor that use electronic switching for the current in the stator winding segment to accomplish commutation. This motor is classified as the synchronous motor with a trapezoidal flux distribution.

The grasping mechanism is conducted by controlling the force when grasping. Therefore we are interested in the speed/torque characteristic performance of the gripper. Since the brushless DC motor is a synchronous motor, first we will discuss the detailed machine formulations for synchronous motor in section 2. Next, the equations considering the brushless DC motor will be presented in section 3, which is given in state space equation. Finally in section 4 the transfer function and the block diagram of the gripper will be given.

2 Mathematical Model and Torque Considerations for the Synchronous Machine

The majority of research associated with the formulation of models for the simulation of synchronous machines is directed towards the synchronous generator. The class of synchronous machines contains several specific machines that are differing with respect to their rotor structures. Therefore the description of each sub-case of the synchronous machine is treated from the general synchronous machine[2].

This section presents the detailed machine formulations for synchronous motors which, in addition to provide the basis for a more advanced study of the subject. It may also be used to advantage for a number of problems associated with practical electro-driving system. Taking as a general example, the synchronous machine where the motor is presented as a 3-phase synchronous machine with the phase winding in a star-connection (wye-connection). All mutuals between stator and rotor circuits are periodical functions of rotor angle position.

In machine variables, the general expression for the voltage equation will involve the symmetric stator phases (designated by subscripts *as*, *bs*, and *cs* for phases *a*, *b*, and *c*) and the symmetric rotor windings (designated by subscripts *kq* and *kd* for the *q*-axis and *d*-axis auxiliary windings and by subscript *fd* for the field winding, which is assumed to be oriented along the *d*-axis)[3]. If *us*, *is*, λ*s*, are respectively the vector of stator voltages, -currents, and –flux linkages, and *ur*, *ir*, λr, are respectively the vector of the rotor voltages, -currents, and –flux linkages, then the voltage equations are written as

$$\begin{bmatrix} u_s \\ u_r \end{bmatrix} = \begin{bmatrix} \mathfrak{R}_s & 0 \\ 0 & \mathfrak{R}_r \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix} + \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} \quad (1)$$

where $\mathfrak{R}_s$ and $\mathfrak{R}_r$ are stator and rotor resistance matrices

$$\mathfrak{R}_s = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \qquad \mathfrak{R}_r = \begin{bmatrix} R_{kq} & 0 & 0 \\ 0 & R_{fd} & 0 \\ 0 & 0 & R_{kd} \end{bmatrix} \quad (2)$$

When denoting any of the above stator and rotor vectors (voltage, current, and flux) by the generic notation fs, fr, respectively, the vector structure will be

$$f_s = \begin{bmatrix} f_{as} & f_{bs} & f_{cs} \end{bmatrix}^T \qquad (3)$$

$$f_r = \begin{bmatrix} f_{kq} & f_{fd} & f_{kd} \end{bmatrix}^T \qquad (4)$$

where stator components are associated with phase winding in machine variables, and rotor components are associated with the two auxiliary winding and the field winding in machine variables.

Assuming magnetic linearity, the flux linkages may be expressed by

$$\begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s(\theta) & \mathbf{L}_m(\theta) \\ \mathbf{L}_m^T(\theta) & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix} \quad (5)$$

with inductances matrices

Assuming magnetic linearity, the flux linkages may be expressed by

$$\mathbf{L}_s(\theta) = \begin{bmatrix} L_s & M_s & M_s \\ M_s & L_s & M_s \\ M_s & M_s & L_s \end{bmatrix} - L_m \begin{bmatrix} \cos(2p\theta) & \cos(2p\theta - \frac{2\pi}{3}) & \cos(2p\theta + \frac{2\pi}{3}) \\ \cos(2p\theta - \frac{2\pi}{3}) & \cos(2p\theta) & \cos(2p\theta + \frac{2\pi}{3}) \\ \cos(2p\theta + \frac{2\pi}{3}) & \cos(2p\theta - \frac{2\pi}{3}) & \cos(2p\theta) \end{bmatrix} \quad (6)$$

$$\mathbf{L}_m(\theta) = \begin{bmatrix} M_q \cos(p\theta) & M_f \cos(p\theta) & M_d \cos(p\theta) \\ M_q \cos(p\theta - \frac{2\pi}{3}) & M_f \cos(p\theta - \frac{2\pi}{3}) & M_d \cos(p\theta - \frac{2\pi}{3}) \\ M_q \cos(p\theta + \frac{2\pi}{3}) & M_f \cos(p\theta + \frac{2\pi}{3}) & M_d \cos(p\theta + \frac{2\pi}{3}) \end{bmatrix} \quad (7)$$

$$\mathbf{L}_r = \begin{bmatrix} L_{qr} & 0 & 0 \\ 0 & L_{fr} & 0 \\ 0 & 0 & L_{dr} \end{bmatrix} \quad (9)$$

where :

Ls	–	(average) stator self inductance,
Ms	–	(average) stator to stator mutual inductance,
Lm	–	stator inductance coefficient that accounts for the rotor salient,
Lqr	–	self inductance of the rotor q-axis auxiliary winding,
Ldr	–	self inductance of the rotor field winding
Lfr	–	self inductance of the rotor field winding,
Mr	–	mutual inductance between the two d-axis rotor winding,
Md, Mq, Mf	–	magnitudes of the angle dependent mutual inductances between stator and the various rotor winding,
P	–	the number of pole pairs.

For the motor mechanical dynamics, the differential equation for the rotor velocity ω is

$$J \frac{d\omega}{dt} = T_e - B\omega - T_L \qquad (9)$$

where J is moment inertia of the rotor, is the speed of the rotor, Te is the electromechanical torque, B is the motor damping and TL is the load torque.

Reference Frame Transformation

The presented model is not in convenient form for determining the steady state conditions needed for achieving constant velocity operation, due to the model's periodical dependence on position. This position dependence can be eliminated by a nonsingular change of variables, which effectively projects the stator variables into a reference frame fixed to the rotor.

To simplify the mathematical description of the synchronous machine and to facilitate a simulation, a two axes transformation of the stator and rotor quantities may be performed such that the basic equations posses constant coefficients. The derived matrix equations can be presented in *d*, *q*, *0*-axes frame [Park R.H., 1929].

$$[U] = [R]\,[I] + [\tfrac{d\omega}{dt}] + p\,[\lambda]\omega \qquad (10)$$

where $[U]$, $[R]$, $[I]$, and $[\lambda]$ are respectively voltage matrix, resistance matrix, current matrix, and flux linkage matrix, which is given by $[L][I]$.

Although it is possible to construct transformation to other frames of reference, these would not eliminate the position dependence due to the asymmetry present in the rotor. Since the asymmetrical rotor windings are presumed to be aligned with the rotor frame of reference, just one transformation matrix is necessary. It transforms circuit variables from the stator windings to the fictious windings which rotate with the rotor, and it is given by

$$K_{sr}(\theta) = \sqrt{2/3}\begin{bmatrix} \cos(p\theta) & \cos(p\theta - 2\pi/3) & \cos(p\theta + 2\pi/3) \\ \sin(p\theta) & \sin(p\theta - 2\pi/3) & \sin(p\theta + 2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \qquad (11)$$

This matrix is orthonormal and, hence, its inverse is equal to its transpose. Formally stated, the considered change of variables is defined by

$$\tilde{f}_s = [f_{qs} \;\; f_{ds} \;\; f_{0s}] \qquad (12)$$

$$\tilde{f}_s = K_{sr}(\theta) f_s \qquad (13)$$

where the tilde represents the stator variables in the new coordinate $d, q, 0$-system.
If the stator windings are star-connected, the sum of the stator currents must always be equal to zero, i.e. $i_{as} + i_{bs} + i_{cs} = 0$. By other words, the reference frame transformation, which is intended to eliminate from the voltage equations, will also satisfy the algebraic current constraint by construction.
Generally the stator voltage equations become

$$(14)$$

$$u_{qs} = R_s i_{qs} + \frac{d\lambda_{qs}}{dt} + p\omega\lambda_{ds} \qquad (15)$$

$$u_{0s} = R_s i_{0s} + \frac{d\lambda_{0s}}{dt} \qquad (16)$$

and the rotor voltage equations become

$$0 = R_{kd} i_{kd} + \frac{d\lambda_{kd}}{dt} \qquad (17)$$

$$0 = R_{kq} i_{kq} + \frac{d\lambda_{kq}}{dt} \qquad (18)$$

$$u_{fd} = R_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt} \qquad (19)$$

The transformed flux linkages become

$$\begin{bmatrix} \tilde{\lambda}_s \\ \lambda_r \end{bmatrix} = \mathbf{L} \begin{bmatrix} \tilde{i}_s \\ i_r \end{bmatrix} \qquad (20)$$

where

$$\mathbf{L} = \begin{bmatrix} 0 & L_s - M_s + \frac{3}{2}L_m & 0 & 0 & \sqrt{\frac{3}{2}}M_f & \sqrt{\frac{3}{2}}M_d \\ L_s - M_s - \frac{3}{2}L_m & 0 & 0 & \sqrt{\frac{3}{2}}M_q & 0 & 0 \\ 0 & 0 & L_s + 2M_s & 0 & 0 & 0 \\ 0 & \sqrt{\frac{3}{2}}M_d & 0 & 0 & M_r & L_{dr} \\ \sqrt{\frac{3}{2}}M_q & 0 & 0 & L_{qr} & 0 & 0 \\ 0 & \sqrt{\frac{3}{2}}M_f & 0 & 0 & L_{fr} & M_r \end{bmatrix} \qquad (21)$$

All dependencies on rotor angle θ is eliminated and the analysis is simplified. However, non-linearity is not entirely eliminated. Note that $i_{0s} = 0$ so that the 0-axis equation could be completely ignored.
Due to the motor asymmetry, the most convenient set of electrical variables for expressing the electrical torque consists of stator current and stator flux. Using these variables the torque expression becomes

$$T_e = \frac{3}{2} p (i_{qs}\lambda_{ds} - i_{ds}\lambda_{qs}) \qquad (22)$$

3 Brushless DC Motor

Brushless DC motor is a special case of the synchronous motor where the rotor field winding is replaced by permanent magnets attached to the surface of a smooth rotor, with auxiliary rotor winding removed[5]. Therefore we have $i_{fd} = constant$, $i_{kq} = i_{kd} = 0$, and $Lm = 0$. In order to simplify the notation, the new coefficients concerning magnet flux and inductance are defined, which are stated respectively as follow.

$$\lambda_m = \tfrac{3}{2} M_f i_{fd} \tag{23}$$

$$L = L_s - M_s \tag{24}$$

respectively. Since $Ls = Ms$ the new description allows the torque to be expressed by

$$T_e = p\lambda_m i_{qs} \tag{25}$$

Consider the state vector

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$$

Where $x_1 = \theta$, $x_2 = \omega$, $x_3 = i_{qs}$, $x_4 = i_{ds}$ and the input and output vectors are as follows

$$u = [u_1 \quad u_2]^T \qquad\qquad y = [y_1 \quad y_2]^T$$

where $u_1 = u_{qs}$, $u_2 = u_{ds}$, $y_1 = \theta$, $y_2 = i_{ds}$. Since the motor torque depends only on i_{qs} but not on i_{ds}, the d-axis current is the appropriate choice of the second output. With the above variable assignments, the state space equation can be presented as follow

$$\begin{aligned} \dot{x} &= Fx + Gu \\ y &= Hx + Ju \end{aligned} \tag{26}$$

where

(27)

$$G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \tag{28}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{29}$$

$$J = 0 \tag{30}$$

From above equations, the block diagram for the motor can be presented as the following figure.

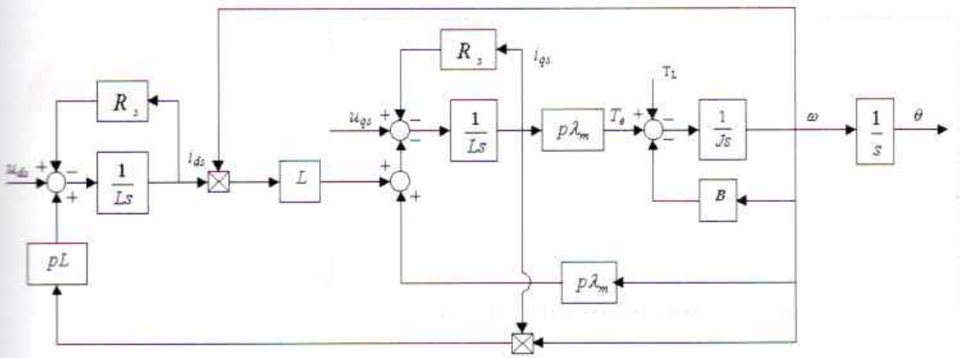

Figure 2 The block diagram of brushless DC motor with d, q domain

From the above block diagram we can see that the electrical Torque (Te) depends only on the iqs. If ids is force to be zero, all we have is the right hand part of the motor model, which lie in the dash rectangle. Since the ids is the output of our state space equation, it can be controlled to be zero.

4 Gripper Model

From Figure 2, we can see that if ids is forced to zero, we will obtain the lock diagram which lies in the dash area. This block diagram is nothing else but the electromechanical model of the common DC motor. One more time the block diagram is illustrated in the following figure.

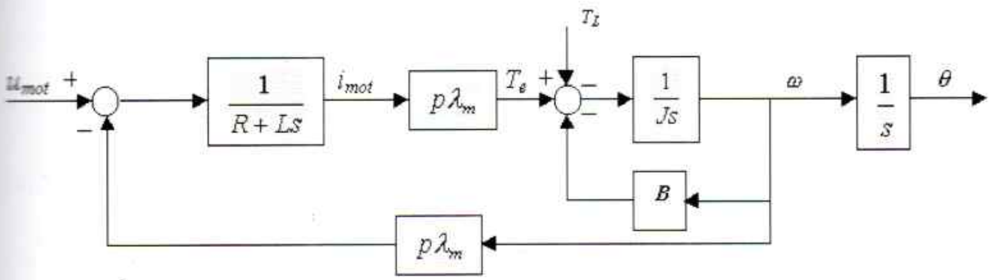

Figure 3 The simplification of the brushless DC motor, where the i_{ds} is equal to zero. The i_{qs} becomes the motor current i_{mot} and the u_{qs} becomes the motor input voltage u_{mot}

Since the value of the *Ls* is very small compared to the resistance (*R*), the transfer function *1/(Ls+R)* is reduced to *1/R*. Thus the equation for i_{mot} becomes

$$i_{mot} = \frac{u_{mot} - p\lambda_m \omega}{R} \qquad (31)$$

PWM Servo Amplifier

The built-in PWM servo amplifier provides current, which is related to the motor current. Since i_{mot} is the only current needs to drive the electrical torque (Te), we can imply that the actual current from the PWM can be referenced as i_{mot}. The current provided by the PWM is proportional to the PWM input voltage with a transconductance gain of *1A/1V* input[4].

The motor current is determined by the input voltage of the amplifier. Therefore we have

$$i_{mot} = g_{mot} u_{in} \qquad (32)$$

where g_{mot} is the amplifier transconductance gain, and u_{in} is the input voltage of the amplifier. From the block diagram above, the equation of u_{mot} is

$$u_{mot} = i_{mot} R + p\lambda_m \omega \qquad (33)$$

Let us express $k_e = p\lambda_m$. Equating the Eq.(32) to Eq.(33), we obtain following equation

$$u_{mot} = g_{mot} R u_{in} + k_e \omega \qquad (34)$$

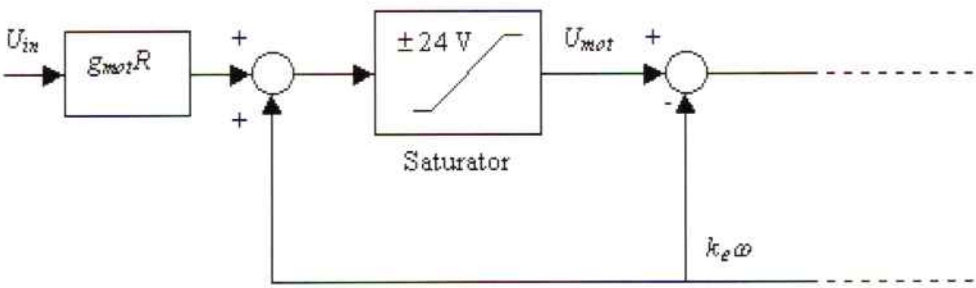

Figure 4 The gripper amplifier model

The maximum voltage allowed for the motor is 24 V (see appendix 3). Because of this saturation we have:

$$u_{mot} = g_{mot} R u_{in} + k_e \omega \qquad \text{for } u_{mot} < 24\text{ V} \qquad (35)$$

$$u_{mot} = 24\text{ V} \qquad \text{for } u_{mot} < 24\text{ V}$$

This is clearer with the block diagram shown in figure

Motor Transformation

The transformation of the gripper is split into two parts: the translation from the motor-shaft to the ball-bearing rotation spindle, and from the spindle to the finger-fastening block. The transformation from the motor-shaft to the ball-bearing rotating spindle comes to pass with a belt. The transformation ratio is

$$\frac{\varphi_s}{\varphi_m} = \frac{1}{4.5} \tag{36}$$

And the transformation from the spindle to the finger-opening

$$\frac{D_v}{\varphi_s} = \frac{2}{\pi} \quad \text{mm/rad} \tag{37}$$

The total transformation n from the motor-shaft to the finger opening is

$$n = \frac{D_v}{\varphi_m} = \frac{9}{4\pi} \approx 0.141 \ \text{mm/rad} \tag{38}$$

The Torque Load

Before the gripper grip an object, the force applied by the fingers is zero. After gripping an object there is force which works on the gripper. Because the workspace of the gripper (the finger opening) only about 10 cm maximum, and to simplify the model it is assumed that any object has its own elasticity constant. So, the force that applied by the finger is:

$$F = k_v D \tag{39}$$

where k_v is the spring constant of the object being gripped, and D is the distance difference of the initial and the final position of the finger.

To generate gripforce, the motor has to increase the torque. The load can be modeled as the direct torque load of the motor. Therefore

$$T_L = nF \tag{40}$$

Since the systems of the gripper such as: rotor, belt, spindle, screw, is not frictionless, the torque load becomes larger than that of the Eq.(40). There is a factor that presents the efficiency of the mechanical system. The total load torque becomes

$$T_L = \eta n F \tag{41}$$

The value of the η is not measured yet. Because of the quality of the gripper mechanical part (this gripper is bought ten years ago), it is difficult to measure this value, and therefore it is estimated by 70%. If $k_m = k_e = p\lambda_m$, then the final model of the gripper can be given as illustrated in Figure 5.

In order to design a proper controller, the gripper model is linearized. Therefore the unlinear parts is removed from the model, and the linearized transfer function is:

$$\frac{F_{out}(s)}{U_{in}(s)} = \frac{g_{mot} k_{mot} \eta}{n} \cdot \frac{\dfrac{k_v n^2}{J\eta}}{s^2 + \dfrac{B}{J} s + \dfrac{k_v n^2}{J\eta}} \tag{42}$$

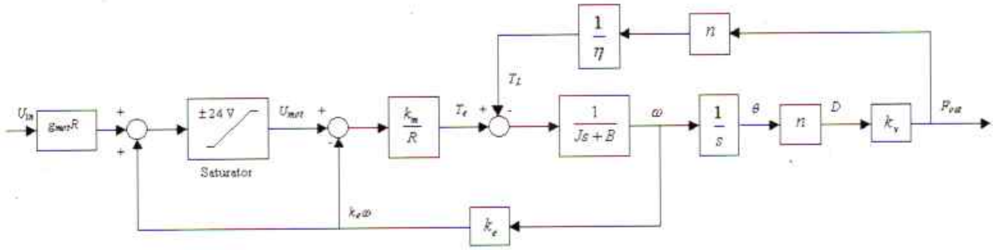

Figure 5 The gripper model

or

$$\frac{F_{out}(s)}{U_{in}(s)} = K_g \frac{\omega_g^2}{s^2 + 2\zeta_g \omega_g s + \omega_g^2} \qquad (43)$$

where

$$K_g = \frac{g_{mot} k_{mot} \eta}{n}$$

$$K_g = \frac{g_{mot} k_{mot} \eta}{n}$$

5 Conclusion

The gripper model later, in much cases is integrated with sensors and actuators. The transfer function of the gripper (equation 43), is integrated with transfer function of sensors, actuators and controllers, and become the transfer function of the grasping system. This gasping system transfer function is used to conduct the simulation of the grasping system. The influence of nonlinearities in the gripper in much cases is compensated by the controller.

6 References

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