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Dissipative Approach in Analysis and Synthesis of Control System via Linear Matrix Inequalities

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Abstract. This paper discusses about application of dissipative concept in dynamical systems for analysis and synthesis of control systems via linear matrix inequalities (LMIs). Motivated by dissipativity concept in mechanical and electrical circuit system, one could employ this concept to check stability and to design a controller of the underlying system. We derive the solvability condition of dissipativity for an output feedback LTI continuous system via LMIs. An algorithm to design such controller that satisfy the dissipativity of the system is also considered.

Keywords ***: dissipative, dissipativity, LMI, solvability condition.***

I. INTRODUCTION

The study of dissipative concept which used to analyze and design of control systems was initially developed by Willems [1]. This concept concerns with analysis and design of control systems using input-output properties based on energy-related description. Practically, a system has the dissipative property if it always dissipates the energy. Dissipation energy is difference between the supply energy and the stored energy in the system. That property includes the notion that a dissipative system never stores more energy than its input energy which come from outside. In the past two decades, there has been a considerable interest in the problems of analysis and synthesis of H_∞ and positive real (or passivity-based) control system. The H_∞ approach builds on the small-gain theorem whereas the positive real approach relies on the positivity theorem. In H_∞ control, the small-gain theorem is used to ensure robust stability by requiring that the loop-gain be less than one at all frequencies. In this scheme, phase information is not used in guaranteeing stability. While, phase information is considered in positivity theory which is widely used in the analysis of passive control system. In the positivity theorem, a (strictly) positive real system has its phase less than 90 degrees so that the loop transfer function of a negative feedback connection of two (strictly) positive real systems has a phase lag of less than 180 degrees. This guarantees stability irrespective of the loop gain. Clearly, both the small-gain and positivity theorems deal with gain and phase performances separately and thus may lead to conservative results in application. A recent paper [2] figures out that the negative feedback interconnection of two causal, stable, linear time-invariant systems, with a "mixed" small gain and passivity property, is guaranteed to be finite-gain stable. However, this lead us to renew dissipative approach which cover both cases.

To be precise, let's take a look two examples in electrical circuit and mechanical system to describe the dissipativity concept. Consider a simple circuit consist of a resistor R, an inductance L, and a capacitor C with current i and voltage u. The differential equation which govern that system is

$$L\frac{dI}{dt} + Ri + V_C = u \qquad (1)$$

The stored energy in the circuit is

$$E(i, V_C) = \frac{1}{2}Li^2 + \frac{1}{2}CV_C^2 \qquad (2)$$

The time derivative of equation (2) which states the rate of the energy when system evolves is

$$\frac{d}{dt}E(i, V_C) = Li\frac{di}{dt} + CV_C\frac{dV_C}{dt} \qquad (3)$$

Inserting the differential equation of the circuit (1) into (3) we get

$$\frac{d}{dt}E(i, V_C) = ui - Ri^2 \qquad (4)$$

Integration of equation (4) from t = 0 to t = T gives

$$E\big(i(T), V_c(T)\big) = E\big(i(0), V_c(0)\big) + \int_0^T u(t)i(t)\,dt - \int_0^T Ri^2(t)\,dt \qquad (5)$$

That equation means that energy at time t = T is the initial energy plus the energy supplied to the system by the voltage u minus the energy dissipated by the resistor. Note that if the input voltage u is zero, and if there is no resistance, then energy E(.) of the system is constant. Here $R \geq 0$ and $E[i(0), Vc(0)] > 0$, and it follows that the integral of the voltage u and the current I satisfies

$$\int_0^T u(t)i(t)\,dt \geq -E\big(i(0), V_c(0)\big) \qquad (6)$$

The physical interpretation of inequality (6) could be seen from the equivalent inequality

$$-\int_0^T u(t)i(t)\,dt \leq E\big(i(0), V_c(0)\big) \qquad (7)$$

which shows that the energy $-\int_0^T u(t)i(t)\,dt$ that can be extracted from the system is less than or equal to the initial energy stored in the system. Another example is borrowed from a simple mechanical system. Consider a one dimensional mechanical system with a mass, a spring, and a damper. The equation of motion for small oscillation of the mechanical system about its equilibrium configuration is

$$m\ddot{x} + D\dot{x} + Kx = F(t);\; x(0) = x_0\,, \dot{x}(0) = \dot{x}_0 \qquad (8)$$

where m is the mass, D is the damper constant, K is the spring stiffness, x is the position of the mass and F is the force acting on the mass. The energy of the system is the sum of its kinetic energy and its potential energy, that is

$$E\big(x(t), \dot{x}(t)\big) = \frac{1}{2}m\dot{x}^2(t) + \frac{1}{2}Kx^2(t) \qquad (9)$$

The rate of change of the energy of the system is

$$L\frac{di}{dt} + Ri + V_c = u \qquad (1)$$

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The rate of change of the energy of the system is

$$\frac{d}{dt}E\big(x(t),\dot{x}(t)\big) = m\dot{x}(t)\ddot{x}(t) + Kx(t)\dot{x}(t) \quad (10)$$

Substituting the equation of the motion (8) into (10) yield

$$\frac{d}{dt}E\big(x(t),\dot{x}(t)\big) = F\dot{x}(t) - D\dot{x}^2(t) \quad (11)$$

Integration of that equation from t = 0 to t = T gives

$$E\big(x(T),\dot{x}(T)\big) = E\big(x(0),\dot{x}(0)\big) + \int_0^T F(t)\dot{x}(t)\,dt - \int_0^T D\dot{x}^2(t)\,dt \qquad (12)$$

Similar to energy equation in the electrical circuit we could interpret that the energy at time t = T is the initial energy plus the energy supplied to the system by the control force F minus the energy dissipated by the damper. Note that if the input F equals to zero, and if there is no damping, then the energy E(.) of the system is constant. Here D ≥0 and E[x(0),

$$\int_0^T F(t)v(t) \geq -E\big(x(0),v(0)\big) \quad (13)$$

The physical interpretation of inequality (13) could be drawn from the equivalent inequality

$$-\int_0^T F(t)v(t) \leq E\big(x(0),v(0)\big) \quad (14)$$

which shows that the energy $-\int_0^T F(t)v(t)\,dt$ that can be extracted from the system is less than or equal to the initial energy stored in the system.

By observing those examples, as seen in equation (4) and (11), one concludes that a dissipative system could be characterized by the power balance equation which states that the rate of change of the energy of the system is equal to the power input injected into the system minus the rate of dissipation energy in the system. Since in real system, dissipation energy always happen, the rate change of the energy of the system always less than or equal to power supply into the system. Using mathematical abstraction of the notions of physical power and energy, researchers have developed the stability analysis and designed the controller for various applications in dissipative systems framework. To name a few, Gupta [3] employed that concept to derive robust stabilization of uncertain systems, Moreno [4] designed observers for a class of nonlinear systems via dissipative method, Stain [5] proposed the dissipative concept for analyze of interconnected oscillators, and Lim, et. al. [6] applied a (non-smooth) dissipative framework for analysis of linear parameter-varying system. The recent paper by Willems [7] introduces dissipativity in the setting of behavioral system.

II. PRELIMINARIES

Consider a dynamic system in state space form $\dot{x} = f(x,u,t)$ $= g(x, u, t)$, where x denotes the system state, u represents input to the system, y is the system output, and two functions f and g describe the system dynamics. This system is said to be dissipative, according to [1], if there exists an absolutely integrable function of the input and the output, the power function $p(u,y)$ (referred to as the supply rate in [1]) and a function of the system state, the energy-like function $V(x) \geq 0$ (referred to as the storage function in [1]) such that

$$V(x(t)) \leq V(x(0)) + \int_0^t p(u(s), y(s))\, ds \qquad (15)$$

holds along all possible trajectories of the system, starting at $x(0)$, for all $x(0)$, $t \geq 0$, or equivalently : for all admissible controllers u(.) that drive the state from $x(0)$ to $x(t)$ on the interval $[0, t]$. In differential form, (15) can be written as follows

$$\dot{V}(x) \leq p(u(t), y(t))$$

Equation (16) stipulates that the rate of change of the stored energy is less than or equal to the input power, the difference being the rate of the energy dissipation. The key property of a dissipative dynamical system is that the total energy stored in the system decreases with time. In this case, there exists an intimately link to Lyapunov stability. Willems [7] said that the notion of a dissipative system is a natural generalization of a Lyapunov function to open systems. A difference between two approaches is that the state of the system and the equilibrium point are notions that required in Lyapunov approach, while the dissipative approach is rather based on the input-output behavior of the plant. The concept of dissipativity is closely connected to that of a storage function. These functions provide convenient Lyapunov functions in stability analysis of the system. Moreover, it has been shown for linear time-invariant systems [1] that dissipativity is equivalent to existence of a storage function. Concerning with Lyapunov stability, equation (16) is a condition for time derivative of Lyapunov function if we set the supply rate $p(u,y)$ to be zero. Thus, analysis dissipativity was begun with find a storage function (as a Lyapunov function candidate) which satisfy (16) with respect to a certain supply rate.

In H_∞-control problem, or more general terminology finite L_2-gain [11], its performance measure is assigned by system norm $\|G(s)\|_\infty \leq \gamma$ which satisfy

$$\int_0^T y(t)^T y(t)\, dt \leq \gamma^2 \int_0^T u(t)^T u(t)\, dt \qquad (17)$$

or all $T \in [0, \infty)$. Thus, these systems is dissipative with respect to the supply rate

$p(u, y) = \gamma^2 u(t)^T u(t) - y(t)^T y(t)$

While, in passive system are characterized by the input output property

$$\int_0^T y(t)^T u(t)\, dt \geq 0 \quad (18)$$

for all $T \in [0, \infty)$. This condition corresponds to dissipativity with respect to $p(u, y) = y(t)^T u(t) + u(t)^T y(t)$.

III. LMI FORMULATION FOR DISSIPATIVE CONTROL

Consider the LTI continuous time system as the following form

$$G:\begin{cases}\dot{x} = Ax + Bu \\ y = Cx + Du\end{cases}$$

where $x \in \Re^n$ is state vector, $u \in \Re^p$ and $y \in \Re^q$ are input and output, respectively. We will assume that A, B, C, and D are of suitable dimensions. Next, consider quadratic supply rate function of the form

$$p(y,u) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} \qquad (20)$$

where dimension of matrices Q, N, and R are determined by those of y and u, and where Q and R are symmetric matrices. The following proposition characterizes the dissipativity of an LTI systems.

Proposition 1. Consider the system G given by (19). If there exists a positive definite matrix P which satisfies the following

$$\begin{pmatrix} A^T P + PA & PB - C^T N & C^T\Theta \\ B^T P - N^T C & -R - N^T D - D^T N & D^T\Theta \\ \Theta^T C & \Theta^T D & -I \end{pmatrix} \qquad (21)$$

the system is dissipative and asymptotically stable, by noting that $Q = -\Theta^T\Theta$.

Proof: The proposition can be proved by assign a candidate Lyapunov function $V(x) = x^T P x$. Taking time derivative of that function, substituting the state space of (19) into that form, and employing the dissipation inequality (16), one can get

$$\begin{pmatrix} A^T P - PA & PB \\ B^T P & 0 \end{pmatrix} \quad \begin{pmatrix} C^T QC & C^T QD + C^T N \\ D^T QC + N^T C & D^T QD - N^T D - D^T N + R \end{pmatrix} < 0$$

The last matrix inequality is nothing more than Schur Complement of (21).

Remark : Finite L_2-gain control problem is dissipative with respect to the specific supply rate function $\gamma^2 u^T u - y^T y$ that is the quadratic supply rate in (20) with Q = −I, N = 0, and R = γ^2I, while passive systems are dissipative with respect to the supply rate (20) with R = Q = 0, and N = I. Now, consider the general framework of control synthesis depicted in fig. 1.

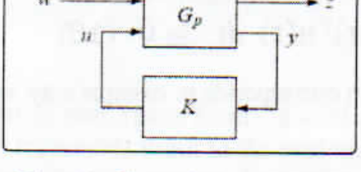

Fig. 1. Closed-loop system

In that figure, G_p is the generalized plant, K is the controller to be designed, w is the disturbance, u is the control input, y is the measurement, and z is the controlled output. We consider an LTI continuous system given in the state space as follows

$$G_p: \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} w + D_{22} u \end{cases} \quad (22)$$

where A, B_i, C_i, and D_{ij} are matrices whose dimensions compatible with x, w, u, z, and y. Dynamics of the controller K is represented by

$$K: \begin{cases} \dot{\xi} = A_K \xi + B_K y \\ u = C_K \xi + D_K y \end{cases} \quad (23)$$

where A_K, B_K, C_K, and D_K are of suitable dimension matrices. The feedback system (22) and (23) is rewritten in state space form as

$$\dot{x}_c = A_c x_c + B_c w$$
$$(24)$$
$$z = C_c x_c + D_c w$$

where

$$A_c = \begin{pmatrix} A + B_2 R D_K C_2 & B_2 R C_K \\ B_K C_2 + B_K D_{22} R D_K C_2 & A_K + B_K D_{22} R C_K \end{pmatrix}$$

$$B_c = \begin{pmatrix} B_1 + B_2 R D_K D_{21} \\ B_K D_{21} - B_K D_{22} R D_K D_{21} \end{pmatrix}$$

$$C_c = (C_1 + D_{12} R D_K C_2 \quad D_{12} R C_K)$$

$$D_c = D_{11} + D_{12} R D_K D_{21}$$

To simplify calculation, we assume, without loss of generality that $D_{22} = 0$. Then the above system matrices can be represented by

$$A_c = A_0 + \hat{B}\Phi\hat{C}$$
$$B_c = B_0 + \hat{B}\Phi\hat{D}_{21}$$
$$C_c = C_0 + \hat{D}_{12}\Phi\hat{C}$$
$$D_c = D_{11} + \hat{D}_{12}\Phi\hat{D}_{21}$$

where

$$A_0 = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}; \hat{B} = \begin{pmatrix} 0 & B_2 \\ I & 0 \end{pmatrix}$$
$$B_0 = \begin{pmatrix} B_1 \\ 0 \end{pmatrix}; \hat{C} = \begin{pmatrix} 0 & I \\ C_2 & 0 \end{pmatrix}$$
$$C_0 = (C_1 \quad 0); \hat{D}_{12} = (0 \quad D_{12})$$
$$\hat{D}_{21} = \begin{pmatrix} 0 \\ D_{21} \end{pmatrix}$$

and

$$\Phi = \begin{pmatrix} A_K & B_K \\ C_K & D_K \end{pmatrix} \qquad (26)$$

To formulate the dissipative control synthesis problem for system (24), consider quadratic supply rate function described by

$$p(w,z) = \begin{bmatrix} z \\ w \end{bmatrix}^T \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} z \\ w \end{bmatrix} \qquad (27)$$

Given the closed-loop system (22) and quadratic supply rate (27), the dissipative control synthesis problem is formulated as follows : *Find controller K of the form (23) such that the closed loop system (24) is asymptotically stable when w = 0 and dissipative with respect to quadratic supply rate (27).*

The feedback system is said to have dissipative performance if there is a positive definite symmetric matrix P which satisfies (21). Now, we will present the main result of this paper in the following theorem.

Theorem 1. Consider the feedback system that is constructed by (22)-(23). Suppose that output feedback dissipative performance control problem formulated above has a solution. Then there exists symmetric positive definite matrices Ψ and Γ such that the following LMIs are satisfied.

$$\begin{pmatrix} N_\Gamma & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} A^T\Gamma - \Gamma A & \Gamma B_1 - C_1^T N & C_1^T\Theta \\ B_1^T - N^T C_1 & -R - N^T D_{11} - D_{11}^T N & D_{11}^T\Theta \\ \Theta^T C_1 & \Theta^T D_{11} & -I \end{pmatrix} \begin{pmatrix} N_\Gamma & 0 \\ 0 & I \end{pmatrix} < 0 \quad (28)$$

$$N_\Psi^T \begin{pmatrix} A\Psi + \Psi A^T & B_1 - \Psi C_1^T N & \Psi C_1 \Theta \\ B_1^T - N^T C_1 \Psi & -R - N^T D_{11} - D_{11}^T N & D_{11}^T\Theta \\ \Theta^T C_1^T \Psi & \Theta^T D_{11} & -I \end{pmatrix} N_\Psi < 0 \quad (29)$$

$$\begin{pmatrix} \Psi & I \\ I & \Gamma \end{pmatrix} > 0 \quad (30)$$

where N_Γ is a matrix whose columns form the bases of null space of $(C_2 \;\; D_{21})$ and N_Ψ is a matrix whose columns form the bases of null space of $(B_2^T \;\; -D_{12}^T N \;\; D_{12}^T\Theta)$.

Proof : Using proposition 1, one can write down the dissipativity condition for the feedback system (22)-(23) with respect to supply rate function (27) is as follows

$$M + \hat{Q}^T\Phi^T\hat{P} + \hat{P}^T\Phi\hat{Q} < 0 \quad (31)$$

where

$$M = \begin{pmatrix} PA_0 + A_0^T P & PB_0 - C_0^T N & C_0^T\Theta \\ B_0^T P - N^T C_0 & -R - N^T D_{11} - D_{11}^T N & D_{11}^T\Theta \\ \Theta^T C_0 & \Theta^T D_{11} & -I \end{pmatrix} \quad (32)$$

$$\hat{P} = \begin{pmatrix} P\hat{B} \\ N^T\hat{D}_{12} \\ \Theta^T\hat{D}_{12} \end{pmatrix} \quad (33)$$

$$\hat{Q} = \begin{pmatrix} \hat{C} & \hat{D}_{21} & 0 \end{pmatrix} \quad (34)$$

Moreover, by employ the elimination of matrix variable (§2.6.2 in [8]), there exists a matrix Φ in (31), if and only if

$$N_{\hat{Q}}^T M N_{\hat{Q}} < 0, \quad N_{\hat{P}}^T M N_{\hat{P}} < 0 \quad (35)$$

holds, where $N_{\hat{Q}}$ and $N_{\hat{P}}$ are matrices whose columns form the bases of null space of $\hat{Q}$ and $\hat{P}$, respectively. Partitioning P and its inverse as follows

$$P = \begin{pmatrix} \Gamma & \Pi \\ \Pi^T & * \end{pmatrix}, P^{-1} = \begin{pmatrix} \Psi & \Upsilon \\ \Upsilon^T & * \end{pmatrix} \quad (36)$$

where Υ and Ψ are $n \times n$ symmetric matrices, it can be easily shown that the first inequality of (35) is equivalent to (28).

On the other hand, it can also be readily established that the condition $N_{\hat{P}}^T M N_{\hat{P}} < 0$ of (35) is equivalent to (29). Finally, it follows from Lemma 7.2 in [9] that the existence of a matrix $P > 0$ satisfying (36) is equivalent to (30).

Theorem 1 provides a quite general results regarding convex characterization for the dissipative control problem of an LTI system to have a solution. Generality of this theorem is provided by noting that finite L_2-gain and passivity condition are obtained by choosing the appropriate supply rate function in the general results. Specifically, finite L_2-gain are obtained by selecting $R = \gamma^2 I, N = 0$, and $Q = -I$ and setting N_Ψ as

$$\begin{pmatrix} W_1 & 0 \\ 0 & I \\ W_2 & 0 \end{pmatrix}$$

with $(W_1^T \quad W_2^T)^T$ is the bases of null space of $(B_2^T \quad D_{12}^T)$.

While passivity are recovered by choosing $Q = R = 0$ and $N = I$, and setting N_Ψ as

$$\begin{pmatrix} W_1 & 0 \\ W_2 & 0 \\ 0 & I \end{pmatrix}$$

where $(W_1^T \quad W_2^T)^T$ is the bases of null space of $(B_2^T \quad D_{12}^T)$.

When the conditions of Theorem 1 are fulfilled, the computation of a controller that solves the dissipative control problem can be carried out along the lines of the procedure proposed by Gahinet and Apkarian [10]. Assuming that the conditions (28)-(30) are satisfied for some matrices Γ and Ψ, a suitable controller can be found as follows :

- Compute two full-column rank real matrices Π and Υ such that $\Upsilon\Pi^T = I - \Psi\Gamma$
- Find the unique solution $P > 0$ of the linear equation :

$$\begin{pmatrix} \Gamma & I \\ \Pi^T & 0 \end{pmatrix} = P \begin{pmatrix} I & \Psi \\ 0 & \Upsilon^T \end{pmatrix} \quad (37)$$

- With the P matrix, compute the controller parameters A_K, B_K, C_K and D_K by solving the LMI (31).

IV. CONCLUSIONS

This paper describes dissipative approach to analyze and design of control systems by using LMIs. The main result presented here is the solvability condition of dissipativity of an output feedback LTI continuous control system problem. Such problem is solvable if there exist two matrices which satisfy three LMIs in Theorem 1. An algorithm to construct the controller for that problem is also derived.

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